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Supersymmetric D-brane Bound States with B-field and Higher Dimensional Instantons on Noncommutative Geometry

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Abstract

We classify supersymmetric D0-Dp bound states with a non-zero B-field by considering T-dualities of intersecting branes at angles. Especially, we find that the D0-D8 system with the B-field preserves 1/16, 1/8 and 3/16 of supercharges if the B-field satisfies the "(anti-)self-dual" condition in dimension eight. The D0-branes in this system are described by eight dimensional instantons on non-commutative $\mathbf{R}^8$. We also discuss the extended ADHM construction of the eight-dimensional instantons and its deformation by the B-field. The modified ADHM equations admit a sort of the 'fuzzy sphere' (embeddings of $SU(2)$) solution.

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1 Introduction

D-branes in a background of an NS-NS two form B-field have been attracting much interest in the development of string theory. The constant magnetic B-field on the D-brane in particular gives a string theoretical realization of the non-commutative geometry [1, 2, 3] and the world-volume effective theory on it is described by the non-commutative Yang-Mills theory. One of features of the non-commutative Yang-Mills theory is that a singularity of a moduli space of instantons on non-commutative geometry is naturally resolved [4] and we do not encounter the problems of the singularity. So the non-commutative geometry is very helpful tool for considering the resolution of the singularity in the moduli space of instantons.

Four dimensional k instantons of the gauge group $U(N)$ is realized as k D0-branes on N D4-branes in Type IIA string theory. If we turn on the anti-self dual constant B-filed, which preserves 1/4 of supersymmetries, the D0-branes are "resolved" and we can not separate the D0-branes and D4-branes as long as preserving supersymmetry. If we divert to D0-brane theory of the gauge group $U(k)$ with N matter multiplets, moduli space of vacua of the Higgs branch coincides with the moduli space of the instantons and the B-field corresponds to Fayet-Iliopoulos (FI) parameters of the theory. If the FI parameters are non-zero, the system can not enter the Coulomb branch which is described by the separation of the D0 and D4-branes. Therefore, these two pictures are consistent.

On the other hand, these systems are equivalent by string duality to the other brane configurations which include rotated branes at angles. The configuration of the intersecting brane at angles has been applied to the construction of three dimensional Chern-Simons system on the branes [5]. The relation between the non-commutative nature of the Wilson line operator in Chern-Simons theory and the non-commutative geometry is discussed in [6]. So the branes with the B-field gives an alternative understanding of dynamics on the intersecting brane at angles.

In this way, the D-brane bound states with the B-field are very interesting in the context of both brane dynamics and brane world-volume theory. Recently, their systems are discussed from various point of view in [7, 8, 9, 10, 11]. The authors discuss the condition for the B-fields preserving the supersymmetry. However, some cases of the enhancement of supersymmetry was missed unfortunately. In the D0-D8 system, there are the cases preserving 1/16, 1/8 and 3/16 of supercharges, for instance. In this paper, we classify the suitable supersymmetric configuration of the constant B-field by using the results of the supersymmetric intersecting brane at angles [12]. We find that the B-field must satisfy the self-dual condition in the D0-D4 or D0-D8 system. In particular, in the

D0-D8 system, the B-field satisfies the extended "self-dual" conditions given by [13, 14], which associate with the symmetry $Spin(7)$, $Spin(6)$ and $Spin(5)$. We obtain only a part of the components of the "self-dual" B-field from the intersecting brane configuration, but the general value of the B-field, which belongs to a representation of the subgroup $Spin(7)$, $Spin(6)$ and $Spin(5)$ of the eight dimensional rotational group $SO(8)$, certainly preserves 1/16, 1/8 and 3/16 of supersymmetries, respectively.

We also discuss in detail the extended ADHM construction of the eight dimensional "self-dual" instantons when the symmetry group is $Spin(5) \simeq Sp(2)$. If we turn on the constant B-field on the D8-brane, the coordinates gain the non-commutativity. We consider the extended ADHM equations on the non-commutative geometry provided by the B-field in various representations of $Sp(2)$. We show that the non-commutativity of the coordinates modifies the ADHM equations and one of the modified equations admits a 'fuzzy sphere' solution, which is proportional to the generator of the $SU(2)$ algebra. This type of the solution is a significant feature on the eight dimensional instantons. This fact means that the moduli space of the eight dimensional instanton possesses a rich structure and its resolution of the singularity is more complicated than the four dimensional case. The application of the non-commutative geometry to the extended ADHM construction is more useful to understand how the singularity is resolved.

This paper is organized as follows. In section 2, we revisit the classification of the residual supersymmetry in the configuration of the intersecting brane at angles. We treat the case of four angles and increase the number of the angles in sequence. The fractions 1/16, 1/8, 3/16 and 1/4 of the supersymmetries appear in the classification. In next section, we relate the configuration of the brane at angles with the D0-Dp system with the B-field. We find that the supersymmetric configuration of the B-field in the D0-D4 or D0-D8 satisfies the self-dual or extended "self-dual" conditions. The fact means that we can expect the general "self-dual" B-field background on the D8-brane preserves the same supersymmetry as the same as the case of the D0-D4 with the anti-self-dual B-field. We use these "(anti-)self-dual" B-field configurations and discuss the resolution of the eight dimensional instantons, which is regarded as the D0-brane on the D8-brane, on the non-commutative geometry in section 4. We find that the "(anti-)self-dual" B-field modifies the extended ADHM construction of the eight dimensional instantons. We also discuss the solution of the modified ADHM equations. The final section is devoted to discussion and comment.

2 Intersecting brane at angles and residual supersymmetry

We start with a configuration of two intersecting M5-branes at angles in M-theory and discuss residual supersymmetries. Supersymmetries of the intersecting M5-branes are completely classified by [12]. We give an account of their work here because of slightly different notations for the later discussion. They discuss the general rotations by five angles, but we consider only the case up to four angles since we would like to obtain the Dp-Dp' system with a B-field as we will discuss in the next section.

We consider the two intersecting M5-branes whose world-volumes are

$$\mathrm{M5} \quad : \quad (013579)\,, \tag{2.1}$$

$$\mathrm{M5}' \quad : \quad \left(0 \begin{bmatrix}1\\2\end{bmatrix}_{\varphi_1} \begin{bmatrix}3\\4\end{bmatrix}_{\varphi_2} \begin{bmatrix}5\\6\end{bmatrix}_{\varphi_3} \begin{bmatrix}7\\8\end{bmatrix}_{\varphi_4} 9\right), \tag{2.2}$$

where the numbers in parentheses represent world-volume directions and the symbol $\begin{bmatrix}\mu\\\nu\end{bmatrix}_\varphi$ means that the brane wold-volume is tilted in (x^μ, x^ν)-plane by an angle φ. The presence of these two M5-branes gives a constraint on a 32 components Killing spinor ϵ in 11 dimensions.

$$\mathrm{M5} \quad : \quad \Gamma_{013579}\epsilon = \epsilon, \tag{2.3}$$

$$\mathrm{M5}' \quad : \quad R\Gamma_{013579}R^{-1}\epsilon = \epsilon, \tag{2.4}$$

where

$$R = \exp\left\{\pi \sum_{i=1}^{4} \varphi_i \Gamma_{2i-1,2i}\right\} \tag{2.5}$$

is a rotation matrix in the spinor representation and φ_i are four angles in the range of $0 \le \varphi_i < 1$. If some of components of this spinor survive after these projections, supersymmetries are unbroken.

We rewrite the eq. (2.4) by using $R\Gamma_{013579}R^{-1} = R^2\Gamma_{013579}$ and eq. (2.3) into

$$(R^2 - 1)\epsilon = 0. \tag{2.6}$$

Introducing the following diagonalized bases of the gamma matrices

$$\Gamma_{013579} = \mathrm{diag}\left(\mathbf{1}_{16}, -\mathbf{1}_{16}\right), \tag{2.7}$$

$$\Gamma_{1234} = \mathrm{diag}\left(\mathbf{1}_8, -\mathbf{1}_8, \cdots\right), \tag{2.8}$$

$$\Gamma_{1256} = \mathrm{diag}\left(\mathbf{1}_4, -\mathbf{1}_4, \mathbf{1}_4, -\mathbf{1}_4, \cdots\right), \tag{2.9}$$

$$\Gamma_{1278} = \mathrm{diag}\left(\mathbf{1}_2, -\mathbf{1}_2, \mathbf{1}_2, -\mathbf{1}_2, \mathbf{1}_2, -\mathbf{1}_2, \mathbf{1}_2, -\mathbf{1}_2, \cdots\right), \tag{2.10}$$

where dots mean repeating the first 16 diagonal components and $\mathbf{1}_n$ stands for $n \times n$ identity matrix, then $R^2 - 1$ in eq. (2.6) reduces to

$$\begin{aligned}
R^2 - 1 \;=\;& 2R\Gamma_{12} \\
& \times \mathrm{diag}\,(\sin\pi(\varphi_1 - \varphi_2 - \varphi_3 - \varphi_4)\mathbf{1}_2, \sin\pi(\varphi_1 - \varphi_2 - \varphi_3 + \varphi_4)\mathbf{1}_2, \\
& \sin\pi(\varphi_1 - \varphi_2 + \varphi_3 - \varphi_4)\mathbf{1}_2, \sin\pi(\varphi_1 - \varphi_2 + \varphi_3 + \varphi_4)\mathbf{1}_2, \\
& \sin\pi(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)\mathbf{1}_2, \sin\pi(\varphi_1 + \varphi_2 - \varphi_3 + \varphi_4)\mathbf{1}_2, \\
& \sin\pi(\varphi_1 + \varphi_2 + \varphi_3 - \varphi_4)\mathbf{1}_2, \sin\pi(\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4)\mathbf{1}_2, \cdots)\,. \qquad (2.11)
\end{aligned}$$

When we act this operator on the Killing spinor ϵ, some components of ϵ can be non-zero if we choose suitable angles, that is, the supersymmetry remains. We can count the number of supersymmetries of the above M5-brane system as the residual spinor components. To see this, we increase the number of the angles in order setting the others to be zero. Then we classify the possible number of supersymmetries and find the condition for the angles.

We notice that the non-rotated M5-brane necessarily breaks half of the supersymmetries due to the projection of the gamma matrix (2.7). Since the projection of the M5-brane preserves only 16 components of the Killing spinor, it is sufficient to consider the action of (2.11) only on the first 16 components in the following.

One angle

If we set $\varphi_2 = \varphi_3 = \varphi_4 = 0$, all components in (2.11) do not vanish unless $\varphi_1 = 0$, namely, supersymmetry is completely broken in general. In the case of $\varphi_1 = 0$, the M5-branes represents nothing but two parallel M5-branes which preserves 1/2 of the supersymmetric charges.

Two angles

We set $\varphi_3 = \varphi_4 = 0$. Some of elements in (2.11) vanish when

$$\varphi_1 \pm \varphi_2 \in \mathbf{Z}. \qquad (2.12)$$

Here all integer values are not allowed since we restrict the angles to the range of $0 \leq \varphi_i < 1$, but we can choose a suitable integer if possible. This notion apples below. For the condition of (2.12), the number of elements being zero in (2.11) is 8 of 16, that is, the system with the condition (2.12) preserves 1/4 supersymmetry.

Three angles

We set $\varphi_4 = 0$. The condition for preserving supersymmetry is

$$\varphi_1 \pm \varphi_2 \pm \varphi_3 \in \mathbf{Z}. \tag{2.13}$$

In this case, 4 of first 16 elements are zero in (2.11) and 4 components in 32 of the Killing spinor survive. So we have 1/8 of supersymmetries.

Four angles

In this case, there are the various ways to obtain the supersymmetric configuration in contrast with the above cases. One is the similar condition to the other angles, which is

$$\varphi_1 \pm \varphi_2 \pm \varphi_3 \pm \varphi_4 \in \mathbf{Z}. \tag{2.14}$$

This preserves 1/16 supersymmetry.

The second is obtained by setting independently

$$\varphi_i \pm \varphi_j \in \mathbf{Z} \quad \text{and} \quad \varphi_k \pm \varphi_l \in \mathbf{Z}. \tag{2.15}$$

There are 12 conditions in total. For example, if we set $\varphi_1 = -\varphi_2 \neq \varphi_3 = -\varphi_4$, 4 of the first 16 elements in (2.11) vanish. So the supersymmetry is enhanced to 1/8 compared with the previous case.

The final condition is particular. We now set all angles to be the same up to a suitable choice of relative signs, that is,

$$\varphi_1 = \pm\varphi_2 = \pm\varphi_3 = \pm\varphi_4. \tag{2.16}$$

If we choose as $\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4$ for instance, 6 components of the first 16 is zero. So we have 3/16 residual supersymmetries as a result.

3 Supersymmetric D-brane bound states with a B-field

In this section we produce Dp-Dp' bound states, which preserve some supersymmetry, from the M5-brane configuration discussed in section 2 by using the M-theory compactification on $T^8 \times S^1$.

First we consider the compactification on S^1 along the x^9-direction. The M5-brane configuration (2.1) and (2.2) is dual to the intersecting D4-brane at four angles in Type IIA theory. We next would like to take T-dualities along x^1, x^3, x^5 and x^7-directions on the torus T^8 which extends along $x^1, \ldots, x^8$. This T-duality simply maps the M5-brane to the D0-brane, but since the M5′-brane is tilted on T^8 at some angles, this rotated D4′-brane generally produces D8-brane with a B-field on T^8 (See for example [15, 7]) and the geometry on T^8 will be non-commutative. The B-field is given by

$$B_{\mu\nu} = \frac{1}{2\pi\alpha'} \begin{pmatrix} 0 & b_1 & & & \\ -b_1 & 0 & & & \\ & & \ddots & & \\ & & & 0 & b_4 \\ & & & -b_4 & 0 \end{pmatrix}, \tag{3.1}$$

where $\mu, \nu = 1, \ldots, 8$ and b_i are related to the four angles by

$$\tan 2\pi\varphi_i = -\frac{1}{b_i}, \tag{3.2}$$

or equivalently,

$$e^{4\pi i\varphi_i} = -\frac{1+ib_i}{1-ib_i}. \tag{3.3}$$

This D0-D8 bound state preserves the same supersymmetry as the configuration discussed in the previous section since T-duality does not change the number of the supercharges. Of course, we can consider from the beginning the conditions for the Type IIA Killing spinors when the B-field exists on the D0-D8 bound state, but it just give the same conditions as the intersecting brane case. Moreover, there is no distinction of the twisted boundary condition for the worldsheet theory between the intersecting brane at angles and D-brane with the B-field. So we can simply apply the supersymmetric classification for the brane at angles to the D-brane bound state with the B-field by using the translation rule (3.3).

In the classification of the previous section, we set some angles to be zero in the case that the number of the rotated angles is less than four. Since this means that some parts of the worldvolume directions are parallel to M5-brane, T-duality finally makes the D0-D2, D0-D4 and D0-D6 bound states with the B-field, which correspond to the one angle, two angles and three angles case, respectively.

So we can now classify the supersymmetric condition for the B-field in sequence.

D0-D2 system

The one angle case corresponds to the D0-D2 bound state with a constant B-field on the D2. For general value of b_1, the supersymmetry is completely broken even if D0-D2 system without the B-field, which corresponds to the case of $\varphi_1 = 1/4$ and the others are zero. An exception is D0-D2 in the limit of $|b_1| \to \infty$, but this is just equivalent to the D0-D0 system.

D0-D4 system

In this case, the supersymmetric condition of the 1/4 supersymmetry for two angles reduces to

$$\left(\frac{1+ib_1}{1-ib_1}\right)\left(\frac{1+ib_2}{1-ib_2}\right)^{\pm 1} = 1. \tag{3.4}$$

This means that

$$B_{12} = \mp B_{34}, \tag{3.5}$$

that is, the D0-D4 system is supersymmetric if and only if the B-field is anti-self-dual or self-dual.

D0-D6 system

There are various choices of the 1/8 supersymmetric condition for the three angles depending on the relative signs of the angles. We here only consider the case that the signs are all positive without loss of generality. Then the condition for the b_i is

$$\prod_{i=1}^{3}\left(\frac{1+ib_i}{1-ib_i}\right) = -1, \tag{3.6}$$

so we find that

$$B_{12}B_{34} + B_{34}B_{56} + B_{56}B_{12} = 1, \ \cdots. \tag{3.7}$$

It is not clear what this condition means in the sense of the anti-symmetric two form field.

The D0-D6 bound state without the B-field corresponds to the intersecting branes at angles of $\pm\varphi_1 = \pm\varphi_2 = \pm\varphi_3 = \frac{1}{4}$ and $\varphi_4 = 0$. These angles do not satisfy the condition (2.13), so the D0-D6 system can not be supersymmetric if a suitable B-field does not exist.

D0-D8 system

In this case, there are three patterns depending on the fractions of the residual supersymmetry. First, we choose one of the conditions as $\sum_{i=1}^{4}\varphi_i = 0$ which preserves the 1/16 supersymmetry, then the condition for b_i is

$$\prod_{i=1}^{4}\left(\frac{1+ib_i}{1-ib_i}\right) = 1. \tag{3.8}$$

From this, we find that

$$\begin{aligned} &B_{12} + B_{34} + B_{56} + B_{78} \\ &\quad = B_{12}B_{34}B_{56} + B_{34}B_{56}B_{78} + B_{56}B_{78}B_{12} + B_{78}B_{12}B_{34}, \ \cdots. \end{aligned} \tag{3.9}$$

This condition reduces to the simple form in the small B limit as

$$B_{12} + B_{34} + B_{56} + B_{78} = 0, \ \cdots. \tag{3.10}$$

This consists of an extended "self-dual" condition in dimensions greater than four discussed in [13, 14],

$$\frac{1}{2} T_{\mu\nu\rho\sigma} B^{\rho\sigma} = \lambda B_{\mu\nu}, \tag{3.11}$$

where $T_{\mu\nu\rho\sigma}$ is a totally antisymmetric tensor and λ is an eigenvalue. The tensor $T_{\mu\nu\rho\sigma}$ is not invariant under the 8 dimensional rotational group $SO(8)$ but at least must be invariant under subgroups of $SO(8)$. The above condition (3.10) breaks $SO(8)$ to $Spin(7)$ and **28** of $B_{\mu\nu}$ decomposes into **21** which corresponds to $\lambda = 1$ and the tensor is given by

$$T^{\mu\nu\rho\sigma} = \eta^T \gamma^{\mu\nu\rho\sigma} \eta, \tag{3.12}$$

where $\gamma^{\mu\nu\rho\sigma}$ is the totally anti-symmetric product of $SO(8)$ gamma matrices and η is a constant unit spinor, which satisfies $\eta^T \eta = 1$.

The second condition preserving 1/8 of the supersymmetries is typically $\varphi_1 = \varphi_2 \neq \varphi_3 = \varphi_4$. This condition contains two sets of the self-dual condition on each four dimensional submanifolds

$$B_{12} = B_{34}, \ B_{56} = B_{78}, \ \cdots, \tag{3.13}$$

in terms of the B-field. In this case the tensor $T_{\mu\nu\rho\sigma}$ is a singlet under the subgroup $(SU(4) \times U(1))/\mathbf{Z}_2$ and (3.13) is a part of the "self-dual" condition (3.11) with $\lambda = 1$. This type of the condition could be also considered as an Hermitian-Einstein condition

$$g^{\alpha\bar{\beta}} B_{\alpha\bar{\beta}} = 0, \tag{3.14}$$

where $g^{\alpha\bar{\beta}}$ is a hermitian metric on the T^8.

Note that the condition (3.13) or (3.14) is associated with the invariance under $SU(4) \simeq Spin(6)$, which is a holonomy of a Calabi-Yau four-fold (CY_4). This fact means that the intersecting brane at angles of (2.15) or D0-D8 system with the B-field of (3.13) is dual to the CY_4.

Finally, we consider the case preserving 3/16 supersymmetries in the D0-D8 system. One of the conditions is $\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4$. The condition is expressed in terms of the B-field as

$$B_{12} = B_{34} = B_{56} = B_{78}, \ \cdots. \tag{3.15}$$

This also is a kind of the extended "self-dual" condition which is invariant under the subgroup $(Sp(2) \times Sp(1))/\mathbf{Z}_2$. Under this symmetry, the $B_{\mu\nu}$ in the adjoint representation survive as **10** when $\lambda = 1$ and satisfies the condition (3.15)[1]. The tensor $T_{\mu\nu\rho\sigma}$ is a singlet of $Sp(2) \simeq Spin(5)$, which is characteristic symmetry of the D0-D8 system with the B-field of (3.15).

In the construction of the B-field from the branes at angles, we have obtained only a part of the components of the "self-dual" B-field, but we can expect that the general "self-dual" B-field preserves the same supersymmetry since the vev of the B-field is compatible with the holonomy of the dual manifold of the D-brane bound states with the B-field. So we treat the components of the B-field as the general "self-dual" one in the following.

4 D0-D8 system with B-field and extended ADHM construction on noncommutative geometry

4.1 Noncommutative instanton as D0-D4 with the B-field

The ADHM construction [16, 17] is very important tool for finding gauge configurations and analyzing moduli space of instantons. We first briefly review the ADHM construction on four dimensional space and its modification on the noncommutative $\mathbf{R}^4$. The noncommutativity of $\mathbf{R}^4$ is realized by introducing a constant magnetic B-field on the D4-brane. The supersymmetric D0-D4 bound state with the B-field is regarded as a resolved instanton on the noncommutative $\mathbf{R}^4$ whose moduli space is non-singular. We in the following concentrate the case of the gauge group $U(N)$ with instanton number k since it simply relates to the bound state of k D0-branes and N Dp-branes.

We first consider the instantons on the ordinary commutative space. In the ADHM construction, it is useful to treat four coordinates of $\mathbf{R}^4$ as a quaternion. Introducing the quaternionic basis $\sigma_\mu = (i\tau_1, i\tau_2, i\tau_3, \mathbf{1}_2)$, where τ_i are the Pauli matrices, the coordinates are arrenged into the quaternion variable

$$\boldsymbol{x} = \sum_{\mu=1}^{4} \sigma_\mu x^\mu = \begin{pmatrix} z_2 & z_1 \\ -\bar{z}_1 & \bar{z}_2 \end{pmatrix}, \tag{4.1}$$

where we define the complex coordinates as $z_1 = x^2 + ix^1$ and $z_2 = x^4 + ix^3$.

We first define an operator

$$\mathcal{D}_z = \boldsymbol{A} + \boldsymbol{B}\boldsymbol{x}, \tag{4.2}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are $(N + 2k) \times 2k$ matrices and the product with $\boldsymbol{x}$ is defined in the quaternionic sense as we will see concretely below.

[1]Notice that the choice of the coordinates and the normalization of λ differ from [14].

If we can find a solution of the following equations

$$\mathcal{D}_z^\dagger \psi = 0, \tag{4.3}$$

for $(N+2k)\times N$ matrix ψ, which is normalized as $\psi^\dagger\psi = \mathbf{1}_N$, we obtain the $U(N)$ gauge field

$$A_\mu = \psi^\dagger \partial_\mu \psi. \tag{4.4}$$

Using a completeness relation

$$\mathbf{1}_{N+2k} = \psi\psi^\dagger + \mathcal{D}_z(\mathcal{D}_z^\dagger\mathcal{D}_z)^{-1}\mathcal{D}_z^\dagger, \tag{4.5}$$

and assuming that $\mathcal{D}_z^\dagger\mathcal{D}_z$ are invertible and commute with all quaternions, we have the field strength from the gauge field (4.4)

$$\begin{aligned} F_{\mu\nu} &= 2\psi^\dagger \left(\partial_{[\mu}\mathcal{D}_z(\mathcal{D}_z^\dagger\mathcal{D}_z)^{-1}\partial_{\nu]}\mathcal{D}_z^\dagger\right)\psi \\ &= 2\psi^\dagger \boldsymbol{B}\bar{\eta}_{\mu\nu}(\mathcal{D}_z^\dagger\mathcal{D}_z)^{-1}\boldsymbol{B}^\dagger\psi, \end{aligned} \tag{4.6}$$

where $\bar{\eta}_{\mu\nu} = \frac{1}{2}(\sigma_\mu\sigma_\nu^\dagger - \sigma_\nu\sigma_\mu^\dagger)$ is a constant self-dual tensor which satisfies $\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\bar{\eta}^{\rho\sigma} = \bar{\eta}_{\mu\nu}$. Therefore, the field strength automatically satisfies the self-dual condition.

The commutativity of $\mathcal{D}_z^\dagger\mathcal{D}_z$ with quaternions is the crucial condition in order to obtain the self-dual field strength. So all of our tasks finding the self-dual gauge field configuration of instantons amount to solving the commuting conditions for $\mathcal{D}_z^\dagger\mathcal{D}_z$. From the definition of $\mathcal{D}_z$, we have

$$\mathcal{D}_z^\dagger\mathcal{D}_z = \boldsymbol{A}^\dagger\boldsymbol{A} + \boldsymbol{A}^\dagger\boldsymbol{B}\boldsymbol{x} + \boldsymbol{x}^\dagger\boldsymbol{B}^\dagger\boldsymbol{A} + \boldsymbol{x}^\dagger\boldsymbol{B}^\dagger\boldsymbol{B}\boldsymbol{x}, \tag{4.7}$$

then we can rewrite into the condition for the comporments of $\boldsymbol{A}$ and $\boldsymbol{B}$ with respect to each order in $\boldsymbol{x}$ since the commuting condition must satisfy for any $\boldsymbol{x}$.

Before rewriting the condition, we notice that there are equivalences between different sets of $\boldsymbol{A}$ and $\boldsymbol{B}$ as

$$\boldsymbol{A} \sim U\boldsymbol{A}M, \qquad \boldsymbol{B} \sim U\boldsymbol{B}M, \tag{4.8}$$

where $U \in U(N+2k)$ and $M \in GL(2k,\mathbf{C})$. The gauge field is invariant under this transformation. Using this symmetry, we can arrange the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ into

$$\boldsymbol{A} = \begin{pmatrix} A_2 & A_1 \\ -A_1^\dagger & A_2^\dagger \\ I & J \end{pmatrix}, \qquad \boldsymbol{B} = \begin{pmatrix} \mathbf{1}_k & 0 \\ 0 & \mathbf{1}_k \\ 0 & 0 \end{pmatrix}, \tag{4.9}$$

where A_i and B_i are $k \times k$ and I and J are $N \times k$ matrices. Substituting these matrices into (4.7) and requiring the commutativity of $\mathcal{D}_z^\dagger \mathcal{D}_z$ with quaternions, we obtain the following sets of equations

$$\mu_{\mathbf{R}} \equiv [A_2^\dagger, A_2] - [A_1^\dagger, A_1] + I^\dagger I - J^\dagger J = 0, \tag{4.10}$$

$$\mu_{\mathbf{C}} \equiv [A_2^\dagger, A_1] + I^\dagger J = 0. \tag{4.11}$$

These one real and two complex equations are known as the ADHM equation of instantons. Three degrees of the equations associate with **3** of $SU(2) \simeq Sp(1)$, which reflects the hyper-Kähler structure of the instanton moduli space. If we find the solutions of the ADHM equations, we can construct the self-dual field strength through ψ which satisfies eq. (4.3). Therefore, the moduli space of instantons is described by the hyper-Kähler quotient in the space of the solutions

$$\mathcal{M} = \left(\mu_{\mathbf{R}}^{-1}(0) \cap \mu_{\mathbf{C}}^{-1}(0) \right) / U(k). \tag{4.12}$$

This moduli space is singular, but we can resolve the singularity by the modification of the ADHM equations.

Let us next consider the resolution of the ADHM equations by turning on the B-field. In string theory, if the constant B-field exists on the D4-brane the coordinates on the D4-brane become non-commutative

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}. \tag{4.13}$$

$\theta^{\mu\nu}$ are determined by the B-field as

$$\theta^{\mu\nu} = 2\pi\alpha' \left(\frac{1}{g + 2\pi\alpha' B} \right)_A^{\mu\nu}, \tag{4.14}$$

where the subscript A means an anti-symmetric part of the matrix.

The B-field on the D4-brane admits only the self-dual or anti-self-dual one for preserving supersymmetry. However, the self-dual B-field does not change the ADHM equations of the self-dual instanton. Therefore, the self-dual B-field is useless for resolving the moduli space (4.12). However, as we will see below, a constant anti-self-dual B-field deforms the self-dual ADHM equations.

We first introduce the anti-self-dual B-field having only a non-zero component of

$$B_{12} = -B_{34} = \beta_1. \tag{4.15}$$

This non-zero B-field gives the non-commutativity on $\mathbf{R}^4$ in the complex coordinates

$$[z_1, \bar{z}_1] = -[z_2, \bar{z}_2] = -\frac{\zeta}{2}, \tag{4.16}$$

and others are zero. Here we define $\frac{\zeta}{4} = -\frac{2\pi\alpha'\beta_1}{1+\beta_1^2}$.

From the commuting condition of the operator $\mathcal{D}_z^\dagger D_z$, we find the modification of the first ADHM equation (4.10)

$$\mu_{\mathbf{R}} = \zeta. \tag{4.17}$$

The second is not changed.

We can also introduce the following non-zero components of the B-field with keeping the anti-self-dual condition

$$B_{14} = -B_{23} = \beta_2, \qquad B_{13} = B_{24} = \beta_3. \tag{4.18}$$

This gives

$$[z_1, z_2] = -\rho, \tag{4.19}$$

where $\rho = -2(\beta_2 - i\beta_3)/\Delta$ and $\Delta = 1 + \beta_1^2 + \beta_2^2 + \beta_3^2$. These non-commutative coordinates modify the condition for the ADHM construction of the instantons and we have in addition to (4.17)

$$\mu_{\mathbf{C}} = \rho. \tag{4.20}$$

As a result, the singularity of the moduli space of instantons on the commutative $\mathbf{R}^4$ is resolved to a smooth manifold

$$\tilde{\mathcal{M}} = \left(\mu_{\mathbf{R}}^{-1}(\zeta) \cap \mu_{\mathbf{C}}^{-1}(\rho)\right) / U(k). \tag{4.21}$$

Here we see the constant anti-self-dual B-field modifies the ADHM equations of the self-dual instantons. If we would like to consider the ADHM construction of the anti-self-dual instantons, one may exchange a role of σ_μ and $\sigma_\mu^\dagger$. Then, we have a anti-self-dual tensor $\eta_{\mu\nu} = \frac{1}{2}(\sigma_\mu^\dagger \sigma_\nu - \sigma_\nu^\dagger \sigma_\mu)$ in the formula (4.6) instead of $\bar{\eta}_{\mu\nu}$. The self-dual B-field only affects the ADHM equations of the anti-self-dual instantons vice-versa.

4.2 Resolution of D0 in D8 by the B-field

We now consider the extended ADHM construction of the eight dimensional "self-dual" instantons with $Sp(2)$ symmetry given in [18], and its modification by the B-field.

In order to treat the eight dimensional space, we first provide two quaternionic coordinates in which we arrange the eight coordinates as

$$\boldsymbol{x} = \sum_{\mu=1}^{8} \tilde{\sigma}_\mu x^\mu = \begin{pmatrix} z_2 & z_1 \\ -\bar{z}_1 & \bar{z}_2 \end{pmatrix}, \quad \boldsymbol{x}' = \sum_{\mu=1}^{8} \tilde{\sigma}'_\mu x^\mu = \begin{pmatrix} z_4 & z_3 \\ -\bar{z}_3 & \bar{z}_4 \end{pmatrix}, \tag{4.22}$$

where we define the eight vector matrices

$$\begin{aligned}\widetilde{\sigma}_\mu &= (i\tau_1, 0, i\tau_2, 0, i\tau_3, 0, \mathbf{1}_2, 0), \\ \widetilde{\sigma}'_\mu &= (0, i\tau_1, 0, i\tau_2, 0, i\tau_3, 0, \mathbf{1}_2),\end{aligned}$$

and the four complex coordinates $z_1 = x^3 + ix^1$, $z_2 = x^7 + ix^5$, $z_3 = x^4 + ix^2$ and $z_4 = x^8 + ix^6$.

The Dirac-like operator in the ADHM construction extends to

$$\mathcal{D}_z = \boldsymbol{A} + \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{x}}, \tag{4.23}$$

where $\vec{\boldsymbol{B}} = (\boldsymbol{B}, \boldsymbol{B}')$ and $\vec{\boldsymbol{x}} = (\boldsymbol{x}, \boldsymbol{x}')$. The matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{B}'$ are $(N+2k)\times 2k$ similar to the previous case.

The construction of instantons is also similar to the four dimensional case. Finding the solution of the equation (4.3) at first and using the relation

$$\Sigma_\mu \equiv \partial_\mu \vec{\boldsymbol{x}} = \begin{pmatrix} \widetilde{\sigma}_\mu \\ \widetilde{\sigma}'_\mu \end{pmatrix}, \tag{4.24}$$

then we find the "self-dual" field strength

$$\begin{aligned}F_{\mu\nu} &= 2\psi^\dagger \left(\partial_{[\mu}\mathcal{D}_z(\mathcal{D}_z^\dagger\mathcal{D}_z)^{-1}\partial_{\nu]}\mathcal{D}_z^\dagger\right)\psi \\ &= 2\psi^\dagger \vec{\boldsymbol{B}}\bar{N}_{\mu\nu}(\mathcal{D}_z^\dagger\mathcal{D}_z)^{-1}\vec{\boldsymbol{B}}^\dagger\psi,\end{aligned} \tag{4.25}$$

where $\bar{N}_{\mu\nu} = \frac{1}{2}(\Sigma_\mu\Sigma_\nu^\dagger - \Sigma_\nu\Sigma_\mu^\dagger)$ is a "self-dual" tensor which satisfies $\frac{1}{2}T_{\mu\nu\rho\sigma}\bar{N}^{\rho\sigma} = \lambda\bar{N}_{\mu\nu}$ with $\lambda = 1$.

Here we require the condition that $\mathcal{D}_z^\dagger D_z$ should commute with Σ_μ. This is a necessary condition to obtain the "self-dual" gauge configurations on the eight dimensions.

In this construction, there are equivalences between different sets of $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{B}'$ as like as (4.8). Using this symmetry, we can rearranged $\boldsymbol{A}$ and $\boldsymbol{B}$ as in (4.9) and $\boldsymbol{B}'$ as[2]

$$\boldsymbol{B}' = \begin{pmatrix} B_2 & B_1 \\ -B_1^\dagger & B_2^\dagger \\ K & L \end{pmatrix}. \tag{4.26}$$

In this representation of matrices, the commuting condition of $\mathcal{D}_z^\dagger D_z$ gives the following sets of equations

$$\mu^1_{\mathbf{R}} \equiv [A_2^\dagger, A_2] - [A_1^\dagger, A_1] + I^\dagger I - J^\dagger J = 0, \tag{4.27}$$

[2]Precisely speaking, the degrees of freedom of the relative coordinate choices in $\boldsymbol{x}$ and $\boldsymbol{x}'$ still remain, but we have fixed them for later convenience.

$$\begin{aligned}
\mu^1_{\mathbf{C}} &\equiv [A_2^\dagger, A_1] + I^\dagger J = 0, && (4.28)\\
\mu^2_{\mathbf{C}} &\equiv [A_2^\dagger, B_2] - [B_1^\dagger, A_1] + I^\dagger K - L^\dagger J = 0, && (4.29)\\
\mu^{2\,\prime}_{\mathbf{C}} &\equiv [A_2^\dagger, B_1] + [B_2^\dagger, A_1] + I^\dagger L + K^\dagger J = 0, && (4.30)\\
\mu^3_{\mathbf{R}} &\equiv [B_2^\dagger, B_2] - [B_1^\dagger, B_1] + K^\dagger K - L^\dagger L = 0, && (4.31)\\
\mu^3_{\mathbf{C}} &\equiv [B_2^\dagger, B_1] + K^\dagger L = 0. && (4.32)
\end{aligned}$$

There are two real and four complex equations, which relate to the adjoint representation **10** of $Sp(2)$. So we expect that the moduli space of the eight dimensional instantons considering now has a structure of $Sp(2)$ holonomy. An example of the manifold with $Sp(2)$ holonomy is known as the toric hyper-Kähler manifold [19] and a relation between the 't Hooft type solution of the above extended ADHM equations and the manifold with $Sp(2)$ holonomy is discussed in [20].

We next consider the effect of the B-field on the ADHM construction in eight dimensions. In our considering case, the tensor $T^{\mu\nu\rho\sigma}$ in the "(anti-)self-dual" condition is invariant under the subgroup $(Sp(1) \times Sp(2))/\mathbf{Z}_2$ of $SO(8)$. Under this symmetry, **28** of anti-symmetric two form B in eight dimensions decomposes into **10**, **3** and **15**, which satisfy the "(anti-)self-dual" condition (3.11) with $\lambda = 1, -\frac{5}{3}, -\frac{1}{3}$, respectively. Therefore, there are one "self-dual" and two "anti-self-dual" B-field, which are compatible with $Sp(2)$ symmetry, and we may have the modification of the extended ADHM equations by these B-fields.

First, we consider the case that the B-field is **10** with $\lambda = 1$. We obtain the following commutation relations of the complex coordinates from the non-zero components of the B-field

$$\begin{aligned}
&[z_1, \bar{z}_1] = [z_2, \bar{z}_2] = -\frac{\zeta_1}{2}, \qquad [z_3, \bar{z}_3] = [z_4, \bar{z}_4] = -\frac{\zeta_2}{2},\\
&[z_1, z_2] = -\rho_1, \quad [z_1, \bar{z}_3] = [z_2, \bar{z}_4] = -\rho_2,\\
&[z_1, z_4] = -[z_2, z_3] = -\rho_3, \quad [z_3, z_4] = -\rho_4,
\end{aligned}$$

and the others are zero, where $\zeta_i \in \mathbf{R}$ and $\rho_i \in \mathbf{C}$ are parameters which are determined by the components of the B-field. The extended ADHM equations in this background are modified while the B-field is "self-dual" in contrast with the four dimensional case. In fact, if we require the commuting condition of $\mathcal{D}_z^\dagger \mathcal{D}_z$, the first equation (4.27) becomes

$$\mu^1_{\mathbf{R}} = -\bar{\rho}_2 B_2 + \rho_2 B_2^\dagger - \bar{\rho}_3 B_1 + \rho_3 B_1^\dagger, \tag{4.33}$$

that is, linear terms in B add to the quadratic ADHM equation. However, unfortunately, the solution of the above equation is not so interesting. The l.h.s and r.h.s of (4.33) must be independently zero since the l.h.s is hermitian but r.h.s is not. If we choose as $B_2 = \rho_2 H_2$ and $B_1 = \rho_3 H_1$, where H_i are hermitian matrices, the equation (4.33) is satisfied. Therefore, the moduli space of the eight dimensional instantons almost is not modified and we can not avoid the singularity. The "self-dual" B-field is not useful for the resolution of the "self-dual" instanton moduli space, consequently.

The next case is **3** with $\lambda = -\frac{5}{3}$. If we define $N_{\mu\nu} = \frac{1}{2}(\Sigma_\mu^\dagger \Sigma_\nu - \Sigma_\nu^\dagger \Sigma_\mu)$, this satisfies the "anti-self-dual" condition with $\lambda = -\frac{5}{3}$. From the three independent components of the B-field, we find the non-commutativity of the complex coordinates

$$
\begin{aligned}
&[z_1, \bar{z}_1] = -[z_2, \bar{z}_2] = [z_3, \bar{z}_3] = -[z_4, \bar{z}_4] = -\frac{\zeta}{2}, \\
&[z_1, \bar{z}_2] = [z_3, \bar{z}_4] = \rho,
\end{aligned}
$$

and the other commutation relations are zero. These commutation relations modify the extended ADHM equations as

$$
\begin{aligned}
\mu^1_{\mathbf{R}} &= \zeta\left(\mathbf{1}_k + \Xi\right), &(4.34)\\
\mu^1_{\mathbf{C}} &= \rho\left(\mathbf{1}_k + \Xi\right), &(4.35)
\end{aligned}
$$

where $\Xi = \frac{1}{2}\left(\{B_2^\dagger, B_2\} + \{B_1^\dagger, B_1\} + K^\dagger K + L^\dagger L\right)$. This is very similar to the four dimensional instanton except for the existence of Ξ. In particular, the first two equations are the same as the resolved ordinary ADHM equation when $B_1 = B_2 = 0$ and $K = L = 0$, namely the moduli space of the eight dimensional instanton includes the resolved four dimensional instanton moduli space.

Finally, we consider most interesting case which has a rich structure. The B-field of **15** obeys the "anti-self-dual" condition with $\lambda = -\frac{1}{3}$ and has 15 independent components. The 15 components are arranged into one real parameter ζ and seven complex parameters ρ_i and we have the following commutation relations

$$
\begin{aligned}
&[z_1, \bar{z}_1] = -[z_2, \bar{z}_2] = -[z_3, \bar{z}_3] = [z_4, \bar{z}_4] = -\frac{\zeta}{2}, \\
&[z_1, \bar{z}_2] = -[z_3, \bar{z}_4] = \rho_1, \quad [z_1, \bar{z}_3] = -[z_2, \bar{z}_4] = -\rho_2, \\
&[z_1, z_4] = [z_2, z_3] = \rho_3, \quad [z_2, \bar{z}_3] = -\rho_4, \quad [z_1, \bar{z}_4] = \rho_5, \\
&[z_2, z_4] = \rho_6, \quad [z_1, z_3] = \rho_7.
\end{aligned}
$$

These commutation relations gives the modification of the ADHM equations

$$
\begin{aligned}
\mu^1_{\mathbf{R}} &= \zeta(\mathbf{1}_k - \Xi) + \bar{\rho}_2 B_2 + \rho_2 B_2^\dagger - \bar{\rho}_3 B_1 - \rho_3 B_1^\dagger, &(4.36)\\
\mu^1_{\mathbf{C}} &= \rho_1(\mathbf{1}_k - \Xi) - \bar{\rho}_4 B_2 + \rho_5 B_2^\dagger + \bar{\rho}_6 B_1 - \rho_7 B_1^\dagger. &(4.37)
\end{aligned}
$$

In this case we have linear terms in B_i again. However, in contrast with the previous case, there are non-trivial solutions proportional to k dimensional representation of $SU(2)$ algebra, $T_\pm = T_1 \pm iT_2$ and T_3. Actually, if we tune the parameters to $\zeta = \rho_1 = 0$ and $\rho_2 = \rho_3 = 2\rho_4 = 2\rho_5 = 2\rho_6 = 2\rho_7 = m$, then we find a solution

$$\begin{aligned} &A_2 = mT_-, \quad A_1 = mT_-, \quad B_2 = mT_3, \quad B_1 = mT_3, \\ &I = m\lambda_1, \quad J = m\lambda_2, \quad K = m\lambda_3, \quad L = m\lambda_4, \end{aligned}$$

where λ_i are $N \times k$ matrices which satisfy that $\lambda_1^\dagger \lambda_4 = \lambda_2^\dagger \lambda_3 = T_3$, $\lambda_1^\dagger \lambda_1 = \lambda_2^\dagger \lambda_2$, $\lambda_3^\dagger \lambda_3 = \lambda_4^\dagger \lambda_4$ and the other $\lambda_i^\dagger \lambda_j$ $(i \neq j)$ are zero. For example, when $N = 4$ and $k = 2$ if we choose $\lambda_1 = -i\Sigma_5$, $\lambda_2 = -i\Sigma_6$, $\lambda_3 = \Sigma_8$ and $\lambda_4 = \Sigma_7$, all equations are satisfied. The existence of these solutions means that the moduli space of the eight dimensional instantons is divided into disconnected pieces every representation of $SU(2)$.

From a point of view of the D0-brane world-volume theory, the matrices A_i and B_i correspond to the adjoint vector multiplets of the gauge group $U(k)$, and I, J, K, L are N matter multiplets. The ADHM equations discussed above are flat conditions which describe the moduli space of vacua in D0-brane world-volume theory. If we add the B-field background, the flat conditions are changed. One of the modifications appears as the FI parameters as like as the ordinary supersymmetric Yang-Mills theory of D0-D4 system. However, the linear term in B_i means the mass term in the system. So the B-field also provides mass of the vector multiplets. Similar modifications of the flat direction appear in the context of supersymmetric Chern-Simons theory [21, 22, 23, 24], the dielectric effect of the branes [25] and the $\mathcal{N}$=1* theory [26]. Presumably, the brane realization of these systems may relate to the D0-D8 with the B-field by the string dualities.

5 Discussion and Comment

In this paper, we discussed the supersymmetric configuration of the D0-Dp system with the B-field. We obtain the supersymmetric system by using the T-dualities from the intersecting branes at four angles. In the classification of the supersymmetric intersecting M5-branes at angles [12], we can maximally rotate the brane by up to five angles. The rotation by five angles includes 1/32, 3/32 and 5/32 supersymmetric cases in addition to what we have discussed. However, if we naively apply the dimensional reduction and the T-dualities to these configurations in M-theory, we obtain a bound state of D0 and p D8-brane with a B-field and q rotated NS5-branes at angles, where p and q are co-prime integers determined by the fifth angle. This system also preservers the same supersymmetries as the intersecting branes, but the fractions of the supersymmetry sound strange in a sense of the D-brane bound states with the B-field. To understand this system we may need more knowledge about the brane dynamics near the NS5-brane [27].

The brane systems with the B-field is considered as a dual to the other brane configurations. The low energy effective theories on these branes are described by non-commutative Yang-Mills theory, Chern-Simons theory, ordinary Yang-Mills theory with non-zero FI parameters, etc. The dynamics of these theories are closely related with each other by dualities in string theory. The careful analyses of the D-brane bound states with the B-field and their duals will shed light on the problems of various field theories.

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References

[1] A. Connes, M. R. Douglas and A. Schwarz, "Noncommutative Geometry and Matrix Theory: Compactification on Tori", JHEP **9802** (1998) 003, `hep-th/9711162`.

[2] M. R. Douglas and C. Hull, "D-branes and the Noncommutative Torus", JHEP **9802** (1998) 008, `hep-th/9711165`.

[3] N. Seiberg and E. Witten, "String Theory and Noncommutative Geometry", JHEP **9909** (1999) 032, `hep-th/9908142`.

[4] N. Nekrasov and A. Schwarz, "Instantons on noncommutative $\mathbf{R}^4$, and (2,0) superconformal six dimensional theory", Comm. Math. Phys. **198** (1998) 689-703, `hep-th/9802068`.

[5] T. Kitao, K. Ohta and N. Ohta, "Three-Dimensional Gauge Dynamics from Brane Configurations with (p,q)-Fivebrane", Nucl. Phys. **B539** (1999) 79-106, `hep-th/9808111`.

[6] K. Ohta, "Supersymmetric Index and s-rule for Type IIB Branes", JHEP **9910** (1999) 006, `hep-th/9908120`.

[7] B. Chen, H. Itoyama, T. Matsuo and K. Murakami, "$p{-}p'$ System with B-field, Branes at Angles and Noncommutative Geometry", Nucl. Phys. **B576** (2000) 177-195, `hep-th/9910263`.

[8] M. Mihailescu, I.Y. Park and T.A. Tran, "D-branes as Solitons of an N=1, D=10 Non-commutative Gauge Theory", `hep-th/0011079`.

[9] E. Witten, "BPS Bound States Of D0-D6 And D0-D8 Systems In A B-Field", `hep-th/0012054`.

[10] R. Blumenhagen, V. Braun and R. Helling, "Bound States of D($2p$)-D0 Systems and Supersymmetric p-Cycles", `hep-th/0012157`.

[11] Y. Imamura, "A D2-brane realization of Maxwell-Chern-Simons-Higgs systems", `hep-th/0012254`.

[12] N. Ohta and P. K. Townsend, "Supersymmetry of M-Branes at Angles", Phys. Lett. **B418** (1998) 77-84, `hep-th/9710129`.

[13] E. Corrigan, C. Devchand, D .B. Fairlie and J. Nuyts, "First-order Equations for Gauge Fields in Space of Dimension Greater than Four", Nucl. Phys. **B214** (1983) 452-464.

[14] R. S. Ward, "Completely Solvable Gauge-field Equations In Dimension Greater Than Four", Nucl. Phys. **B236** (1984) 381-396.

[15] J. Polchinski, "String Theory", Vol. II, Cambridge University Press (1998).

[16] M. Atiyah, N. Hitchin, V. Drinfeld and Y. Manin, "Construction of Instantons", Phys. Lett. **65B** (1978) 185.

[17] E. Corrigan and P. Goddard, "Construction of Instanton and Monopole Solutions and Reciprocity", Ann. Phys. **154** (1984) 253-279.

[18] E. Corrigan, P. Goddard and A. Kent, "Some Comments on the ADHM Construction in $4k$ Dimensions", Comm. Math. Phys. **100** (1985) 1-13.

[19] J.P. Gauntlett, G.W. Gibbons, G. Papadopoulos and P.K. Townsend, "Hyper-Kahler manifolds and multiply-intersecting branes", Nucl. Phys. **B500** (1997) 133-162, `hep-th/9702202`.

[20] G. Papadopoulos and A. Teschendorff, "Instantons at Angles", Phys. Lett. **B419** (1998) 115-122, `hep-th/9708116`.

[21] G. Dunne, "Self-Dual Chern-Simons Theories", `hep-th/9410065`.

[22] H.-C. Kao and K. Lee, "Self-Dual Chern-Simons Higgs Systems with an N=3 Extended Supersymmetry", Phys. Rev. **D46** (1992) 4691-4697, `hep-th/9205115`.

[23] H.-C. Kao, "Self-Dual Yang-Mills Chern-Simons Higgs Systems with an N=3 Extended Supersymmetry", Phys. Rev. **D50** (1994) 2881-2892.

[24] K. Ohta, "Moduli Space of Vacua of Supersymmetric Chern-Simons Theories and Type IIB Branes", JHEP **9906** (1999) 025, `hep-th/9904118`.

[25] R.C. Myers, "Dielectric-Branes", JHEP **9912** (1999) 022, `hep-th/9910053`.

[26] J. Polchinski and M. J. Strassler, "The String Dual of a Confining Four-Dimensional Gauge Theory", `hep-th/0003136`.

[27] S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici and G. Sarkissian, "D-Branes in the Background of NS Fivebranes", JHEP **0008** (2000) 046, `hep-th/0005052`.